

NAME: _____

AI2 Final Exam Fall 2025

Each question is worth 10 points.

1. There is about 0.05% chance of having a certain disease¹⁹. About 80% of the people with the disease experience symptoms of high fever and cough. About 5% of those that have symptoms end up testing positive. Joe wakes up one morning with a high fever and cough. What is the probability that Joe will test positive for disease¹⁹?
2. We run a bank's lending department. From past data, 15% of loans end up bad. Our loan application process is very detailed: *we record thousands of data points on each applicant*. After crunching past loans, we discover that bad loans were to people with mustaches 60% of the time (good loans involve a mustache only 10% of the time). A customer with a mustache walks in: use the Bayes rule to get probability that the customer will default on their loan:
3. We codify the rule from previous question, and put it into production. After sometime, we discover that the default rate remains mostly unchanged. What could have gone wrong?
4. We next use income brackets: from past data, 70% of loan defaults are in \$1-40k income bracket, 20% of loan defaults are in \$40-100k income bracket, 10% are in \$100-and-up bracket. 60% of good loans (non-defaulted) are from \$1-40k income bracket, 35% from \$40-100k bracket, and 5% from \$100-and-up. A customer with \$50k income walks in, according to Bayes rule what's the probability of default?
5. Another feature that appears useful is whether the applicant has a car loan. We notice that 90% of defaulted applicants also had a car loan, while only 25% of non-default applicants had a car loan. A customer with a car loan walks in, according to Bayes rule what's the probability of default?

6. Being very clever, we first apply the income bracket model, followed by the car-loan check model. A customer with \$50k income walks in, with an existing car loan, according to Bayes rule what's the probability of default?
7. Continuing from previous question, using Naive Bayes assumption, what's the probability of default after observing income bracket and car-loan feature?
8. Consider a 2-layer neural network with 2 inputs, 2 neurons in the hidden layer, and 1 output neuron. The network uses the ReLU activation function, defined as $\text{ReLU}(x) = \max(0, x)$. The network weights and biases are given as:

$$\text{Hidden layer weights: } W_1 = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{Output layer weights: } W_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad b_2 = 2$$

The input to the network is:

$$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Calculate the final output of the network. Show work (calculate the output of the hidden layer, apply ReLU, etc.)

9. Fair coin flipping game: We start with \$1. Heads we win 50%, tails we lose 50%. After 3 rounds, with a fair coin, the *arithmetic mean* value we will have:
10. Fair coin flipping game. We start with \$1. Heads we win 50%, tails we lose 50%. After 3 rounds, with a fair coin, the *median* value we will have: